A multifractal approach for extracting relevant textural areas in satellite meteorological images

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Abstract

In the latest years, the use of computer vision tools for automatically analyzing the large amount of data acquired by remote sensing has grown in importance and number of different applications, ranging from basic research to industry. However, images displaying natural phenomena, and especially turbulence, develop following complicated patterns which are difficult to segment and to analyze with those tools. In this paper, we discuss on the use of new image processing methods to describe complicated flow and flow-like quantities, in applications such as meteorology. Using infrared satellite images as an example, we show that we are naturally led to gain insight in the physical and geometrical properties of the observed complex structures. We analyze different processing techniques (multiscale texture classification and multifractal decomposition and reconstruction) issued from the so-called multiscale methodology. The efficiency of multiscale methodology lies on its ability of reproducing known, experimental physical properties of the systems in study (such as scale invariance or multiscaling exponents) in the analysis scheme of images. We show that this methodology can be further exploited in order to derive information about a dynamical property from still infrared images. Namely, the main goal of our study is to detect and characterize textural areas at which typical convective movements take place. For that purpose, we compare the actual graylevel distribution in images, providing information about the temperature distribution, and a synthetic graylevel distribution induced by the multifractal formalism, also reinterpreted by its connection with thermodynamics. The conclusions of our work can be generalized to any analogous physical system.

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1. Introduction

In a number of scientific domains, signals that display flow and flow-like quantities have to be analyzed. Among them, environmental sciences (oceanography and meteorology) provide typical examples where fluid phenomena are involved (Frisch, 1995; Arrault et al., 1997; Tkalich, in press). In this context, the observation of the earth from space results in a large variety of images carrying information about the dynamics of atmosphere. For instance, infrared (IR) images acquired by geostationary satellites provide a large coverage of complex meteorological structures (Arnaud et al., 1992). Thus, it has given rise to a growing interest in image processing tools and their applications on satellite images (Feidas, 2003).

Segmentation of meteorological satellite images has been approached by many authors using computer vision techniques, such as for identification and classification of cloud systems (Ebert, 1989; Kittler and Pairman, 1985) or determination of cloud activity (Lakshmanan et al., 2000; Papin et al., 2000).
In particular, techniques exploiting textural information have given rise to an abundant literature (Baraldi and Parmigiani, 1995; Gu et al., 1991; Welch et al., 1988). However, those algorithms are efficient in identifying and classifying large-scale cloudy structures (Brewer et al., 1997; Pankiewicz, 1996) but they show difficulties to discriminate small-scale cloudy areas that can appear relevant in weather prediction (Lakshmanan et al., 2000; Peak and Tag, 1994). More generally, classical image processing operators show very poor results in tasks such as pattern recognition, segmentation, etc. when applied to images of phenomena governed by fluid mechanics (Corpetti et al., 2002; Grazzini et al., 2002). When dealing with meteorological data, like IR images, one is mainly confronted with the complex inner structure related to the turbulent character of the atmospheric fluid (Frisch, 1995). Precisely, IR images (10.5–12.5 μm wavelength) carry measures of a thermodynamical variable, the temperature, on higher layers of atmosphere, which follow a complicated pattern (Roux et al., 2000). For climatic purposes, like rainfall estimation, the meaningful entities are generally located in the most chaotic areas, and, thus, are difficult to segment (Arnaud et al., 1992; Jobard and Desbois, 1993; Papin et al., 2000). Moreover, satellite data are often acquired in noisy conditions, including numerous artefacts overlapping on the turbulent region (sharp variations in luminance, missing data,...). Taking into account the turbulent nature of those data, appropriate techniques should enable to mimic the properties of the meteorological processes. One of these techniques is that of multifractal formalism, which provides a multiscale description of images (Arrault et al., 1997; Roux et al., 2000; Grazzini et al., 2002).

Multiscale techniques have been proved in the past to be useful to characterize scale free phenomena as the rain, concentrating on its time evolution (Nakken, 1999). In this paper we propose to extract cloudy textural entities relevant for rainfall estimation by performing a multifractal analysis on IR images. Such images allow to access both geometrical (spatial organization, structure) and physical (temperature, radiance) information about the observed phenomena (Arnaud et al., 1992; Jobard and Desbois, 1993). Thus, we adopt a multifractal approach, derived from thermodynamical concepts, that combines aspects from the physics and statistics of IR signals to process them. Namely, due to the fact that atmosphere is a flow in Fully Developed Turbulence (Frisch, 1995), we expect any intensive physical quantity to define a multifractal structure (Parisi and Frisch, 1985; She and Levêque, 1994). This way, IR images can be hierarchically decomposed, from sharp edges to softer textures, into fractal sets with relevant statistical and physical properties (Grazzini et al., 2002; Turiel and Parga, 2000). In this decomposition, a particular fractal set, that mainly corresponds to the strongest transitions of the signal, enables the description of the flow dynamics and retains the most meaningful features of the image. As a consequence, it enables to retrieve the original image through a rather simple and efficient multifractal synthesis algorithm (Turiel and del Pozo, 2002). The determination of this set is the starting point for the analysis of the underlying meteorological phenomena displayed in IR images. Precisely, we develop and evaluate a new approach for characterizing convective areas on still IR images. For this purpose, we introduce a particular synthetic image reconstructed from that latter fractal set, with the same geometrical structure as the original image but with a different temperature distribution. This synthetic image corresponds to the reduced situation where the flow would be only advective. For this reason, it is used to identify textural areas in images where advective motion is in default, i.e. cloudy structures with vertical (convective) movement. Such areas, when they exist, are of strong importance in weather forecasting, as they are mainly responsible for precipitations (Arnaud et al., 1992; Jobard and Desbois, 1993), but they have an elusive nature from IR acquisitions (Papin et al., 2000; Feidas, 2003). The final aim of the study would be to localize rainfalls (Laughlin et al., 2003), which are related with convective transport, but we will only partially discuss that issue.

The paper is organized as follows. In Section 2, we review the multifractal formalism for multiscale characterization of turbulence and chaotic information in digital images. We present the multifractal model to hierarchically decompose IR images into fractal sets and we discuss the relevance of these sets for the knowledge of the underlying phenomena. Then, we introduce the so-called most singular manifold, that mainly consists in the multiscale edges of the image and we show how this set determines the complex inner structure of the image, from both statistical and physical points of view. As an illustration, we show, in Section 3, how it could explain the poor results provided by classical textural methods. For this purpose, we generalize, from examples from the literature, a representation based on cooccurrence features in a multiscale framework and we present the main limitation of this approach over IR images. Back to the multifractal model, we introduce, in Section 4, the multifractal synthesis scheme and the notion of reduced image. We then discuss the characterization of cloudy convective areas, for which we are led to compare, in an appropriate manner, the original IR image and its reduced counterpart provided by the synthesis scheme. We validate those results with other satellite acquisitions displaying rain areas over the same scenes. To conclude, we present the natural extent and the improvements we should carry out in the model.

2. The multifractal approach for describing the image structure

2.1. The multifractal model and the singularity exponents

Methodologies which are related to the properties of the object under study, specially in the case of natural images, need to be considered. For instance, when dealing with IR images (see Fig. 1), the graylevel intensity is identified with a quantity defined over the atmospheric flow, the temperature. Henceforth, the segmentation of IR images has to be carried out by techniques which take into account the properties of the flow, like scale invariance (Frisch, 1995). Such a technique is provided by the multifractal formalism which allows to
decipher complicated structure (Parisi and Frisch, 1985; Frisch, 1995). Multifractality is a property of turbulent-like systems which is present in very different physical systems (Grazzini et al., 2002; Lee et al., 2003). In particular, the multifractal formalism is well adapted to analyze IR measures, as we expect any meaningful intensive variable (like temperature) to behave in an ergodic way, and even more to define a multifractal structure (Arrault et al., 1997; Roux et al., 2000).

Chaotic signals as IR images can be characterized by means of their singularity fronts, i.e. the set of pixels where the most drastic changes in graylevel value occur. The multifractal formalism enables, in particular, the analysis of these irregular transitions. An appropriate technique was used in Grazzini et al. (2002) to detect and extract the singularities from the chaotic and complex IR signal. In this approach, each pixel in the image is assigned a feature that quantifies the strength of the transition the signal undergoes around itself. In order to detect the singularities of the signal, denoted by \( I \), we consider a measure \( \mu \) defined through its density \( d\mu(x) = dx|\nabla I(x)| \) where \( |\nabla I| \) is the norm of the gradient \( \nabla I \). Thus, the size of a ball \( B_r(x) \) of radius \( r \) centered around the point \( x \) with this measure is simply given by:

\[
\mu(B_r(x)) \equiv \int_{B_r(x)} |\nabla I(y)|dy. \tag{1}
\]

Thus, \( \mu(B_r(x)) \) quantifies the local variability of graylevels around the pixel \( x \). It has been shown in Turiel et al. (1998), Turiel and Parga (2000) that for wide ensembles of images such a measure is multifractal. Namely, the measure \( \mu \) is characterized by local singularity exponents \( h(x) \) according to the equation:

\[
\mu(B_r(x)) \sim \alpha(x) r^{d+\gamma h(x)} \tag{2}
\]

where \( d = 2 \) is the dimension of the space and the coefficient \( \alpha(x) \) is scale independent. Over discretized images we need an interpolation scheme to make a log—log linear regression of eq. (2). For that reason, it is convenient to use wavelet projections (Daubechies, 1992), the utility of which has been recognized in a wide range of remote sensing applications (Nakken, 1999; Amato et al., 2000; Laughlin et al., 2003). The wavelet transform of the measure \( \mu \) with the wavelet \( \Psi \) at the point \( x \) and the scale \( r \) is expressed as (Daubechies, 1992):

\[
T_{\Psi}(\mu(x, r) \equiv \frac{1}{\sqrt{r}} \int \Psi \left( \frac{x - y}{r} \right) |\nabla I(y)|dy. \tag{3}
\]

By considering the integral (Eq. (3)), we realize an interpolation of the measure: it enables to capture the behaviour of \( \mu \) over balls \( B_r \) with non-integer radii \( r \). For multifractal measures, the wavelet transform over an appropriate wavelet also exhibits a multifractal behaviour (Daubechies, 1992; Turiel and Parga, 2000):

\[
T_{\Psi}(\mu(x, r) \sim \alpha_{\Psi}(x) r^{h(x)} \tag{4}
\]

where \( \alpha_{\Psi}(x) \) is another scale independent coefficient. Thus, the exponents \( h(x) \) can be directly computed from the wavelet transforms through a log—log linear regression. The accuracy of the estimation and the range of singularity values obtained may depend on the chosen wavelet. Wavelets used for assessing the value of \( h(x) \) are generally those in the family \((1 + |x|^2)^{-\gamma} \) with \( \gamma > 1 \) (Turiel and Parga, 2000).

2.2. The multifractal hierarchy and the most singular manifold

Whatever the nature of the Meteosat IR images, we can apply the multifractal analysis on these images. This way, we perform a direct test of multifractality, just calculating the local singularity exponents at every pixel through the formula (4). In Fig. 1, we show the grayscale representation of singularities \( h(x) \) obtained across an IR image, for which we verify that the log—log regression throws good regression coefficients for the vast majority of points (in Fig. 2 we present...
As a consequence, we can say that IR images are of multifractal nature, and, thus, can be characterized by the estimated local singularity exponents. The multifractal structure enables a hierarchical spatial decomposition of the image from sharp edges to softer textures (Turiel and Parga, 2000). An image can be decomposed into a collection of different fractal components gathering pixels of similar features $h$:

$$F_h \equiv \{ x | h(x) = h \}.$$  \hspace{1cm} (5)

Each fractal set $F_h$ exhibits the same geometrical structure at different scales. The arising hierarchy enables, in particular, to isolate a meaningful structure, the one gathering the strongest transitions: the so-called MSM (Most Singular Component), denoted by $F_{ms}$, associated to the most singular (i.e. most negative) exponent, denoted by $h_{ms}$. Experimentally, the MSM can be directly retrieved from the distribution of the collection of singularity exponents $h(x)$. In Fig. 2 we show the MSM at a rather coarse resolution ($h_{ms} = -0.31 \pm 0.37$): it consists of more or less well defined contours.

### 2.3. Interpretation of the multifractal structures and the MSM

As transitions in the IR signal are related with physical properties, the intrinsic properties of the flow are naturally captured by the multifractal model. Namely, from thermodynamical concepts (She and Levêque, 1994; Turiel et al., 1998), the fractal components are strongly connected with the dynamics of the flow. In turbulence theory (Castaing, 1996; Frisch, 1995), an expression similar to Eq. (1) defines the local dissipation of energy, and it is supposed to discriminate the complicated structures in which energy is injected and dissipated in the flow. Thus, the decomposition $\{F_h\}$ can be regarded as the geometrical representation of a cascade of energy between the different scales (She and Levêque, 1994). In this representation, the MSM can be interpreted as the set from which energy is injected in the flow to the other fractal sets.

In particular, we know (She and Levêque, 1994; Turiel et al., 1998) that the MSM localizes the main current lines of the atmospheric flow, i.e. the lines tangent to the motion field: they are streamlines. Besides, the extraction of the MSM offers an instantaneous estimate of the velocity field over that set and without any need for processing a sequence of images, what happens with more standard methodologies (Béréziat and Berroir, 2000). Nevertheless notice that with the streamlines we do not have access to the celerity (i.e. modulus of the velocity) nor the orientation, but the information it affords is really very relevant. It turns out that this particular fractal set is significative for both image processing (as the set of strongest transitions of graylevels) and physics (as thermal fronts and streamlines). For that reason, we will see that the obtention of the MSM enables the determination of specific complex turbulent regions (Grazzini et al., 2002). More precisely, we will use the knowledge of that set to retrieve some dynamical information about the flow. However preliminarily, in order to highlight the adequacy of multifractal formalism for the segmentation of IR images, we will point out the limitations of some classical texture analysis methods due to the turbulent nature of the data.

### 3. The textural approach and its limitations

#### 3.1. Textural features and cooccurrence based methods

In the context of remote sensing, textural analysis has been often used for image understanding (Gu et al., 1991; Kittler and Pairman, 1985). For instance, image segmentation based on texture classification can be performed over meteorological images, for which it consists in two stages (Jain and Dubes, 1988): first, features characterizing the texture are extracted, then they are used to determine uniform regions over the
The most common methods for feature extraction rely on a statistical approach because of its simplicity in determining characteristics of textures (Kittler and Pairman, 1985; Welch et al., 1988): the local cooccurrence matrices provide statistical measures that allow to represent the spatial dependencies among graylevel values.

Cooccurrence based methods consist in computing the joint probabilities of graylevel pairs, i.e. the relative frequencies of the observed pairwise graylevels separated by a given displacement vector over a domain of the image. Since co-occurrence matrices cannot be used directly for classification, statistical measures describing their structure are extracted and used as representative features (Chen et al., 1989; Welch et al., 1988). In weather analysis, cooccurrence matrices are generally computed for non-overlapping windows to discriminate characteristic large-scale cloud surfaces (Ebert, 1989; Kittler and Pairman, 1985; Welch et al., 1988). However, the local contents of the image need to be considered for segmentation: local features should be extracted instead of the global ones (Randen and Husoy, 1999; Strand and Taxt, 1994) so that they can detect the continuity of a feature as well as the edges between different regions. Typically, this can be done by computing features from local distributions of graylevel pairs, estimated over windows of predefined size and centered on each pixel of the image (Randen and Husoy, 1999). These methods have been shown to perform best in many cases or, at least, to provide comparable performance than other methods (Randen and Husoy, 1999; Strand and Taxt, 1994). Because texture is resolution dependent, approaches based on cooccurrence matrices then need to be extended in order to acquire features at several scales. In the image processing community, the importance of multiscale analysis has been often noted (Mallat, 1989). In Grazzini et al. (2003) it was proposed to generalize cooccurrence matrices in a multiscale framework by introducing a non-uniform and scale-invariant weighting function in the computation of spatial distribution of graylevel variations. Instead of defining a small window around each pixel $x$, we can consider a rather large window $W_X$ where each graylevel pair $(i, j)$ is assigned a weight so that pairs of pixels further away from the base pixel contribute less. Namely, the joint multiscale probability $\tilde{p}_X(i,j)$ is defined for $x$ as:

$$\tilde{p}_X(i,j) \propto \sum_{x,y \in C} \left| x - \frac{y + y'}{2} \right|^{-\alpha} \delta_{i(y') = j(i(y'))}$$  

(6)

with $C = \{(y,y') \in W_X | y' = y + t\}$ the set of pixels pairs in $W_X$ separated by the displacement vector $t$ and $\alpha = 2$. Due to the scale invariant character of the weighting function introduced in Eq. (6), the result does not in principle depend on the size of the window (although the calculations are limited to finite size windows in order to avoid divergences). For that reason, the computation of the features (Welch et al., 1988) does not depend either on any fixed scale. As shown in Grazzini et al. (2003), this method provides a better spatial localization than the classical methods and reduces the overestimation in feature (see Fig. 3). Moreover, for some particular features (e.g. entropy, energy or contrast), assuming statistical translational invariance, it is possible to consider marginal probabilities $\tilde{p}_X(i) \propto \sum_{y} |x - y|^{-\alpha} \delta_{i(y) = j}$ instead of joint probabilities, leading to features attaining a better performance in spatial localization, significance, computer storage and computation time (Baum et al., 1997; Unser, 1986).

3.2. Analysis in feature space

The computation of many features enables to form a multidimensional representation space which helps to determine the class each pixel of the image belongs to (Jain and Dubes, 1988). To avoid the increase of the computation cost and the decrease of the accuracy of the classification, it is necessary, first, to reduce the number of features and, then, to ensure that they provide non-redundant measures. Deciding which features are the most relevant and how they can be combined into an efficient classification scheme has been the focus of many research efforts (Ebert, 1989; Gu et al., 1991). We propose to perform a classification of Meteosat IR images with textural features through two main steps:

- a selection procedure applied on the representation space retains the most discriminant features. By reducing the number of features that need to be collected, we reduce the cost of classification. From examples taken in the literature (Baraldi and Parmigiani, 1995; Gu et al., 1991), we retain only six features for the representation: entropy, energy, contrast, variance, homogeneity and correlation (Welch et al., 1988), for one displacement vector $t = (1, 1)$ only;

- an extraction procedure projects the features in a reduced space where they are uncorrelated. Precisely, a PCA (Jain and Dubes, 1988) enables to combine the features to form a new representative set under a certain inertia criterium.

In Fig. 4, we display some results of the experiences analyzing the mutual dependencies and the correlation in feature space. From those experiences, it turns out that:

- the features convey redundant measures which can be reduced efficiently (i.e. without loss of information) to one or two features only, as shown the measures of the relative inertia conveyed by the principal components extracted by the PCA scheme (see Fig. 4a);

- the features are not independent, nor functionally, neither spatially, as shown by the conditional distributions of pairs of features displaying a remarkable degree of mutual dependency, together with narrow, uni-valued form (see Fig. 4b).

As a consequence, an (unsupervised) segmentation (Jain and Dubes, 1988) scheme based on textural features would fail to accurately segment IR images. It turns out that segmentation is not so good, and it is not improved when new features are included in the classification procedure. The features are all sensitive to the same property of images. When compared with the spectral information conveyed by the luminance, multiple
feature classification does not provide new meaningful characterization. In Fig. 5, we see that textural features have difficulty to extract small textured areas inside the clouds. We will show later how such areas reveal important properties for the knowledge of the convective activity in the cloud.

3.3. Limitations of the approach

As already pointed out in Ebert (1989) and Gu et al. (1991) for the classification of clouds, segmentation methods based on texture extraction do not produce significative results when applied on IR data. One of the reasons of this failure lies in the fact that such methods assume regularity conditions that are not satisfied by IR images: as seen before, those images are related to thermodynamical properties of a turbulent, chaotic flow. Namely, classical methods do not work so efficiently when applied to IR data because of the multiscale properties characteristic to turbulent flows. The existence of an inner structure, that is associated with the most singular transitions and over which textural features are strongly correlated, may explain the poor results obtained by classification methods based on those features. Let us also note that there is a good spatial correspondence between the multifractal decomposition on IR images and the hierarchy induced by one particular feature, the entropy (Grazzini et al., 2002) (compare Figs. 1 and 3).

When confronted with those methods, the multifractal model takes advantage of localization properties and provides a spatial density function of the cooccurrence matrices (Grazzini et al., 2002). Namely, under the assumption of independence of the distributions of $I$ and $V$, the joint distributions of pairwise gray-levels (the $p_x(i, j)$ of Eq. (6)) can be described by the distributions of the signal and its gradient that is used in the multifractal approach. This assumption can be linked with the hypothesis made in Unser (1986) for computing cooccurrence matrices from marginal distributions of graylevel differences instead of joint distribution of graylevel pairs (the $p_x(i, j)$ can be replaced by the probabilities $p_x(i - j)$). If textural methods were employed, a scale invariant decomposition like that obtained by multifractal segmentation could only be retrieved when the zero limit of different resolutions is taken. Taking into account the multiscale nature of wavelet transforms, the computation of the cooccurrence matrices should be performed at different resolutions. Thus, unlike what is claimed in Baraldi and Parmiggiani (1995), there is not an image-independent methodology for feature selection, and, in particular, classification techniques on feature spaces do not work efficiently for every kind of data disregarding the inner structure of images.

Fig. 3. Examples of multiscale textural features computed from the local marginal multiscale distributions $\tilde{p}_x$ (Eq. (6)) on the IR image of Fig. 1. From top to bottom, from left to right: contrast, energy, homogeneity and entropy (see definitions in Welch et al., 1988); the brighter the pixel, the higher the value of the feature at that point.
Fig. 4. Some results of analysis of the mutual dependencies of selected textural features in the representation space. (a) Results of a PCA scheme performed over six multiscale textural features computed on the IR image of Fig. 1. Top, from left to right: representation of the 1st, 2nd and 3rd principal components. Bottom: inertia conveyed by the different components extracted by the PCA scheme. (b) Conditional distributions of textural features w.r.t. the entropy feature (see Fig. 3). From left to right: contrast, homogeneity and energy. The estimations were established on large samples of images, representing more than $10^6$ data. The features were discretized on 255 values.

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Fig. 5. Results of segmentations performed on the IR image of Fig 1. Left: segmentation obtained with entropy feature only. Right: K-Means classification performed with textural features. No significative improvement is obtained when adding features in the classification scheme.
4. The extension of the multifractal model for segmenting convective areas

4.1. The multifractal synthesis scheme

Let us turn back to the multifractal model and consider the structures exhibited by this approach. From the multifractal theory, we know that the whole multifractal signal can be reconstructed from the MSM (Turiel et al., 1998): using only the information contained by that set, it is possible to predict the value of the temperature field at every point. An algorithm based on statistical properties and supposed to produce a perfect reconstructing from the MSM was proposed in Turiel and del Pozo (2002).

The only data required for the reconstruction are a simple vectorial kernel \( g \) and the values of the gradient \( \nabla I \) of the signal over the MSM. In that sense, the MSM can also be interpreted as the most relevant set in the image (Turiel and Parga, 2000), what has been confirmed using Information Theory principles (Grazzini et al., 2002). Let us denote \( \delta_n \) the density function of the set \( F_n \) which equals 1 over the MSM and 0 elsewhere. Let us also define the so-called essential gradient \( \nabla I_n \equiv \nabla \delta_n \), the gradient restricted to the same set. The reconstruction formula (Turiel and del Pozo, 2002) is given by:

\[
I(x) = g \star \nabla I_n (x)
\]

where \( \star \) means convolution dot-product. The kernel \( g \) is completely defined by five conditions: determinism, linearity, translational invariance, isotropy and power-like power spectrum, and its Fourier transform \( \hat{g} \) has a simple form: \( \hat{g}(f) = i f |f|^2 \) where \( f \) denotes the frequency coordinate, \( |f| \) its norm, and \( i \) is the imaginary unit (Turiel and del Pozo, 2002). The principle is that of a propagation of the values of the signal over the MSM to the whole image. On Fig. 6, we represented the essential gradient for the MSM of Fig. 2 and the image reconstructed from it.

4.2. The advective hypothesis and the Reduced Multifractal Image

When dealing with large-scale phenomena in the atmosphere (scales of a few tens of kilometer or more), one usually appreciates several elements that configure their dynamics: almost 2D motion and vertical stratification. Transport in atmosphere is dominated by advection, and is quasi-horizontal: local changes at any point can be explained by advection. However, when vertical motion is present, this hypothesis

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**Fig. 6.** Top: representation of the essential gradient \( \nabla I_n \) over the original image of the Fig. 1 (left) and reconstruction from it (right, PSNR = 30.6 dB). Bottom: orientated MSM (left) whose pixels are assigned a non-white graylevel according to the orientation of the gradients over this set (grey: negative; black: positive) and RMI. In both cases, the MSM used is the one displayed in Fig. 2.
does not hold since ascent or subsidence is associated also with a local tendency (Arnaud et al., 1992). There are a number of different physical mechanisms by which the flow may not be contained in the surface of the image (Arnaud et al., 1992), among them, the vertical transport associated to convec-
tive cloudy structures. To accommodate to real situations, we should consider the properties of the MSM. Namely, the forms of the structures in the image are mainly explained by the shear nature of the flow, i.e. by a variability of the velocity in the direction perpendicular to the flow (let us recall that the MSM represents the main current lines, see Section 2.3).

When the flow is horizontally stratified, the temperature (i.e. the quantity measured by the graylevel intensity) is advected by this flow, and so it moves inside the surface defined by the image. As a consequence, temperature is constant along the MSM. In terms of gradient, the conservation of the intensity \( I \) implies that the gradient \( \nabla I \) is perpendicular to the MSM and of constant modulus. Thus, we can build the field \( \nabla I_R \) associated to advective motion by assigning to every pixel in the MSM a unitary vector, perpendicular to the MSM and (for convenience) with the same orientation as the original gradient \( \nabla I \) (Grazzini et al., 2002). Thus, substituting \( \nabla I_R \) by this rather naive vector field in Eq. (7), we are able to synthetize an image, the so-called Reduced Multifractal Image (RMI, denoted \( I_R \)), that corresponds to the ideal situation where the flow is advective:

\[
I_R(x) = g \star \nabla I_R(x).
\]

Consequently, the RMI has the same multifractal structure (i.e. the same MSM) as the original image, but a more uniform temperature distribution. In fact, such an image ideally attains the most uniform distribution of luminance compatible with the multifractal structure of the original image. We should moreover notice that the RMI defines a more compressed code than the fully reconstructed image. The results of the analysis performed on the IR image of the Fig. 1 are displayed in Fig. 6, where the reduced image is reconstructed from the naive field \( \nabla I_R \) defined on the MSM of Fig. 2.

4.3. Comparison between the original image and the RMI

Focusing our attention on the previous remarks, we should be able to detect the presence of vertical transport in cloudy structures (Arnaud et al., 1992). The advective hypothesis will fail when the flow is not contained in the surface of the image, i.e. essentially for vertical (or convective) movements. Thus, a simple way to assess if the advective assumption holds (resp., if convective structures are present) in a given area is to verify if the gradient along the MSM is (resp., is far from being) constant and perpendicular to the MSM, i.e. if the original image and the RMI coincide (resp., differ). In Fig. 7, we can compare some excerpts of the IR image and the corresponding RMI over areas dominated by advective or convective motion. A visual inspection shows that over advective-dominated areas (essentially clear-sky and low clouds), the image and its reduced counterpart coincide. On the contrary, over areas dominated by vertical motions (the presence of convection is discussed below), the variations between both images are rather large.

We would like to give a more quantitative criterion to measure the differences between both images. For this purpose, the most natural approach consists in measuring the deviation between the fields \( \nabla I \) and \( \nabla I_R \) through the \( L^2 \)-norm of their difference:

\[
\delta(x) = \| \nabla I(x) - \nabla I_R(x) \|_2.
\]

The measure \( \delta \) quantifies how the observed transport \( \nabla I \) differs from the normal propagation, when the quantity \( I \) is approximately advected by the flow, i.e. from \( \nabla I_R \). In Fig. 8, we are able to distinguish, inside the cloudy structure, two main textural areas with different behaviours of \( \delta \). Namely, the center of the cloud is characterized by low \( \delta \)-values, surrounded by a well-defined boundary where \( \delta \)-values become higher. We identify this area with the place at which vertical transport occurs. From the inner boundary to the rest of the cloud, the \( \delta \)-values (and thus the deviation between both fields) are rather high, the highest \( \delta \)-values being attained on the cloud’s edges.

In order to validate the role of the measure \( \delta \), we compare these results with microwave (MW) acquisitions (passive im-
ger onboard the satellite TRMM) acquired over the same scene (Simpson et al., 1996). On these images (see Fig. 8), we are able to detect the so-called convective towers. Namely, cloudy structures like those displayed on the IR image of Fig. 1 (acquired during the rain season) are known to be mesoscale convective clouds, characterized by strong local vertical movements (the towers) that are the site of rainfalls. Precisely, the MW data provide a direct measure of the interaction between the rain at the top of the clouds and the atmosphere. Taking into account the time-lag between both IR and MW images (which represents a few tens of pixels), we see, in Fig. 8, that the texture with low \( \delta \)-values is in rather good correspondence with potentially rainy areas. At the boundaries of the convective towers (with high \( \delta \)-values), the deviation between the hypothetical advective field and the actual field becomes very large, and this deviation is propa-
gated to the rest of the cloud. It is the place where the motion becomes dominated by vertical movements induced by tem-
perature drop or increase. Similarly, the strong \( \delta \)-values observed on the cloud edges are mainly due to the fact that dif-
ferent layers of the atmosphere are measured by the IR signal. Finally, we can say the measure \( \delta \) conveys significative infor-
mation about the nature of the flow. The comparison of the fields through this measure enables to retrieve the main tex-
tural areas associated with vertical transport.

5. Conclusion

In this article, we have first reviewed different multiscale methods for image analysis of data in turbulent flows (such as atmospheric flows) and their connections with thermody-
namical concepts. We have then particularized to a novel, mul-
iscale approach on images that we have applied to the study
of infrared satellite images displaying complicated meteorological structures. Our method is useful to characterize dynamical properties (such as precipitation) which are difficult to observe from such data.

Infrared images are difficult to manipulate due to their intrinsically chaotic character, consequence of the extreme turbulent regime of the atmosphere. For instance, the detection of specific textural areas inside cloudy structures is rather impossible with

Fig. 8. Left: representation of the measure $\delta$ between the fields $\nabla I$ and $\nabla I_R$ (from the lower — black — to the greater values — white). Right: MW image displaying rain areas in dark (the black strips mean no data) over which we draw the IR cloud edges and the contours of the areas with low $\delta$-values. Note that there is a time-lag of 9 min between IR and MW acquisitions.

Fig. 7. Comparison, on two excerpts (with same size) of the Fig. 1, between the real situation displayed on the IR image and the hypothetical advective situation represented by the RMI. Left: the flow is dominated by advection, the images are rather similar. Right: convective structures are present, the images are very different. The range of graylevels was expanded in order to show as many details as possible.
classical methods that are not able to capture complex inner structures. In particular, for meteorological prediction purposes, the detection and characterization of typical convective areas need to be achieved with dedicated methods that try to mimic the properties of infrared images. In this context, we propose, first, to extract, at different scales, the subset gathering the main singularities of the image. This subset contains the key information of the infrared image and can be interpreted as the main current lines of the flow. Then, we compute a reduced image, obtained from a trivial gradient distribution on the main current lines and then applying an interpolation (reconstruction) scheme which is deduced from thermodynamical and statistical properties of turbulent flows. The temperature image so obtained (in which geometrical structures are preserved) can be assimilated to an advective field. A comparison between this image and the original one characterizes textures at which temperature drops or increases take place. In this way, we localize convective structures, associated to vertical movements, that are responsible for hard weather situations. The main advantage of this approach lies in the fact that it is related to a physical—statistical model as it combines both geometrical and physical aspects of the data to process them. From our results, it seems that precipitations can be evidenced (although the segmentation process could be refined.)

Current research is geared toward proposing another suitable transformation to quantitavely compare the original and the reduced images. We are thinking on some Radon-Nykovodin derivative between both fields in order to express their comparison in a more accurate and meaningful way. Apart from its importance in meteorology, this method can serve to detect vertical movements in any other turbulent flow.

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